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FREE-MOLECULAR GAS FLOW IN PLANE CHANNELS AND GRIDS

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FREE-MOLECULAR GAS FLOW IN PLANE CHANNELS AND GRIDS

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Discussion of the problem of the free-molecular flow of a rarefied gas in plane channels or through the cells of a grid, assuming that the grid cells and the overall geometric dimensions of the grid are less than the mean free path of gas molecules and that the grid material is ideally heat conducting. The probability of the passage of gas molecules is calculated and the aerodynamic parameters of the grid are determined for the case of diffusive reflection of gas molecules from a grid or channel.

The flow of rarefied gas in plane channels and grids is discussed assuming /129* a free-molecular state. The parameters of the grid (channel) elements and the total geometric dimensions of the grid are assumed to be smaller than the mean free path of the molecules. It is also assumed that the profile of the grid is thin and that the grid material is an ideal heat conductor. The probability of molecule transmission and the aerodynamic characteristics of the grid are computed for the case when there is a diffusion reflection of molecules from the surface of the grid (channel).

1. The free-molecular flow around a plane grid of profiles. Let us consider a plane grid of thin profiles (fig. 1). We direct the y axis along the grid axis and the x axis perpendicular to the grid axis. The stagger angle (the angle between the x axis and the chord of the profile) is designated by γ. Let us assume that the grid moves with a constant velocity U. For convenience we reverse the problem and assume that the grid is stationary and that the flow is incident, i.e., we shall consider the flow using a system of coordinates fixed with respect to the grid. The direction cosines of the angles between the velocity vector and the coordinate axes will be designated by ι_1 and ι_2 respectively.

The distribution function of the velocities of molecules in the incident flow corresponds to the Maxwell equilibrium distribution law at the flow temperature T_∞, i.e., it has the form

[illegible] (1.1)

Here ξ, η are the components of the thermal velocity of the molecules, R is the gas constant.

The number of molecules n (P) which fall on the surface element (fig. 1) of the mesh $d\sigma(P)$ surrounding the point P, per unit time, may be represented as the sum of the number of molecules $N_1(P)$ of the primary (incident) flow and the number of molecules $N_2(P)$, falling on the element $d\sigma(P)$ due to reflection from another profile of the mesh.

[1] Moscow

* Numbers given in margin indicate pagination in original foreign text.

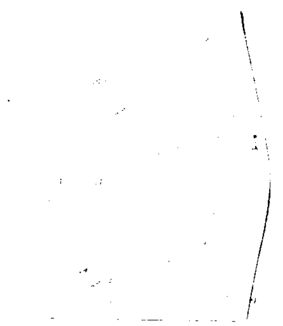

Figure 1. Schematic of the plane grid of profiles.

The quantity $N_1+(P)$ for the upper profile is given by the equation

$$\text{[illegible]} \exp\{-h \ldots \text{[illegible]}\} \qquad (1.2)$$

Assuming that the emission of molecules during diffusion reflection takes place in accordance with Lambert's law, we obtain an expression $N_2+(P)$

$$\ldots = \frac{h}{\pi}\cos\gamma \int d\ldots \int \frac{\ldots\eta^2}{[l + (x_0 - x)\,\mathrm{tg}\,\gamma]^2}\left[\cos\gamma + \frac{x_0 - x}{l + (x_0 - x)\,\mathrm{tg}\,\gamma}\sin\gamma\right] \times$$
$$\times\, n^-(x)\exp\left\{-h\eta^2\left[\left(\frac{x_0 - x}{l + (x_0 - x)\,\mathrm{tg}\,\gamma}\cos\gamma - \sin\gamma\right)^2 + \left(\cos\gamma + \frac{x_0 - x}{l + (x_0 - x)\,\mathrm{tg}\,\gamma}\sin\gamma\right)^2\right]\right\} d\eta \quad (v' = b\cos\gamma) \qquad (1.3)$$

(We note that the expression for N_1+, N_1^- used in references 1 and 2 is inaccurate since the shadow effect over the considered point is not taken into account.[1]

In the same manner we determine $N_1^-(x_0)$, $N_2^-(x_0)$.

Thus the problem is reduced to the solution of a system of two linear integral equations /130

(1.4)

2. Aerodynamic characteristics. After determining the number of molecules n(P) falling on the grid elements we proceed with the determination of aerodynamic forces.

The force acting on the grid may also be represented in the following form:

(2.1)

[1] Computed data on the probability that the gas molecules will pass through the channel are presented in reference 3. The authors do not show an explicit expression for N_1 and N_2.

The reaction force of molecules in the incident flow which have fallen directly on the upper profile (without initial reflections), is determined by means of equations which can be obtained if the expression under the integral sign in (1.2) is multiplied respectively by

$$m\xi_i, \quad m\xi_i \frac{x_0 \operatorname{tg}\gamma - y}{x_0}\Big| \text{ for}$$

where the subscript i refers to the incident molecules. The values $X^-_{0i}(x_0)$, $Y^-_{0i}(x_0)$ are determined in the same way.

The reaction of molecules which have fallen on the upper profile after one or more reflections may be obtained if we multiply the expression under the integral sign in (1.3) respectively by

$$\Big| \text{for}$$

The values $X^-_{1i}(x_0)$, $Y^-_{1i}(x_0)$ are determined in the same way.

Now let us determine the aerodynamic forces which occur during the reflection of molecules (wall emission)

$$\qquad (2.2)$$

$$\qquad (2.3)$$

The values $X^-_r(x_0)$, $Y_r(x_0)$ are determined in the same way.

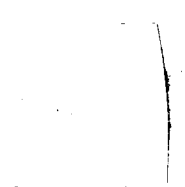

Figure 2. Sketch of the grid mesh (Channel).

If we assume that we have diffusion reflection the coefficient h contained in the above equations depends on the wall temperature. If we assume that the profile material is an ideal heat conductor, i.e., that the temperature over the entire profile is the same, we can formulate the heat balance equation without taking into account the external heat radiation

$$\qquad (2.4)$$

Here ϵ is a coefficient which depends on the properties of the surface, σ is the Stefan-Boltzmann constant, S^* is the reduced radiation area.

Since part of the thermal radiation again falls on the surface of the grid mesh (fig. 2) we obtain the following equation if we assume that energy is radiated according to the Lambert law

(2.5)

The energy transmitted to the upper profile by the incident gas is determined by means of the following equation

(2.6)

The value E_i^- is determined in the same manner.

As we know (ref. 4) the energy removed by the reflected molecules is equal to /131

(2.7)

From the law of the conservation of mass it follows that

3. The flow of rarefied gas through a plane channel. In this case U=0. For simplicity we shall assume that the gas is on one side of the channel while vacuum is on its other side. We determine the probability that the gas molecule will pass through the channel. It is obvious that $n^+(x_0)=n^-(x_0)$. Then system (1.3) is transformed into a single equation

(3.1)

The number of molecules leaving the channel through the cross section CD is equal to $N_-=N_0-N$, where N_0 is the number of molecules which have entered the channel and N is the number of molecules which have left through the cross section AB. The probability that the molecules will pass through the channel is equal to

(3.2)

Equation (3.1) was solved by replacement with a system of algebraic equations. The integral in (3.1) is replaced using the Gauss equation with 4 nodes for approximate integration. The function $\omega(b/t)$ for the values b/t=0.1, 1, 10 is shown in figure 3.

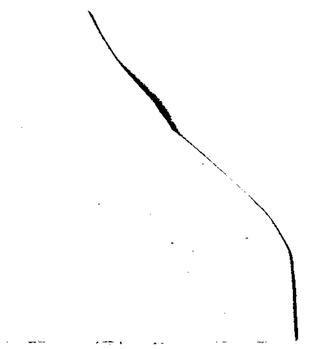

Figure 3. The probability that the molecules have passed through the channel as a function of b/t.

4. The flow of rarefied gas around a plane grid of plates. For simplicity we shall assume that $\gamma=0$ (i.e. the grid has no stagger) and consider the case of high velocities $|U| > C_i = \sqrt{2RT_\infty}$. Then (1.3) assumes the form

[illegible] (4.1)

Now we determine the forces

[illegible] (4.2)

From the conditions of the problem it follows that [illegible]. Equation (2.4) assumes the form ($\gamma=c_p/c_v$)

[illegible] (4.3)

The problem was computed by b/t=0.1, 1, 10.

[illegible]

Received 13 May 1965

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